LECTURE-07

Bernoulli's differential equation

The general form of Bernoulli's DE is

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n$$

Example

(I) $\frac{dy}{dx} + x^2 y = e^x y^2$

(II) $x\frac{dy}{dx} + y = (x+1)\,y^{-1}$

How to solve ?

Let us consider the Bernoulli's DE

$$\frac{dy}{dx} + P(x)\,y = Q(x)\,y^n \quad \text{——(1)}$$

STEP:-1

Dividing both sides by y^n

$$y^{-n}\frac{dy}{dx} + P(x)\,y^{1-n} = Q(x) \quad \text{——(2)}$$

STEP:-2

Put $y^{1-n} = z$

Diff. both sides w.r. to x,

$$\Rightarrow (1-n)\,y^{-n}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow y^{-n}\frac{dy}{dx} = \frac{1}{1-n}\frac{dz}{dx}$$

STEP:-3

Putting these values in equation (2)

$$\frac{1}{1-n}\frac{dz}{dx} + P(x)\,z = Q(x)$$

Scanned by CamScanner

STEP-4

Multiplying both sides by $(1-n)$

$$\frac{dz}{dx} + (1-n)P(x)z = Q(x) \quad \text{——(3)}$$

STEP-5

Which is a linear DE. solve it by linear DE method. Find out the value of z.

STEP:-6

Finally Put $z = y^{1-n}$.

PAGE-39 E. KREYSZIG (8TH EDITION)

31, 32, 33, 36, 37, 38

31. $y' + 2y = y^2$

$\Rightarrow \frac{dy}{dx} + 2y = y^2$ ——— (1)

which is a Bernoulli's DE. $P(x) = 2$, $Q(x) = 1$

STEP:-1

Dividing both sides by y^2

$$y^{-2}\frac{dy}{dx} + 2y^{-1} = 1 \quad \text{——(2)}$$

STEP:-2

Put $y^{-1} = z$, Diff. both sides w.r. to x

$$\Rightarrow -y^{-2}\frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow y^{-2}\frac{dy}{dx} = -\frac{dz}{dx}$$

STEP:-3

Putting these values in equation (2)

$$-\frac{dz}{dx} + 2z = 1$$

$$\Rightarrow \frac{dz}{dx} - 2z = -1 \quad \text{——} (3)$$

Scanned by CamScanner

STEP:-4

which is a linear DE.

So, the integrating factor is $e^{\int -2dx} = e^{-2x}$

STEP:-5

Multiplying the I.F into both sides of equation (3)

$$\frac{dz}{dx} e^{-2x} - 2e^{-2x} z = -e^{-2x}$$

$$\Rightarrow \frac{d}{dx}(ze^{-2x}) = -e^{-2x}$$

$$\Rightarrow d(ze^{-2x}) = -e^{-2x} dx$$

Integrating both sides,

$$\int d(ze^{-2x}) = -\int e^{-2x} dx$$

$$\Rightarrow ze^{-2x} = -\frac{e^{-2x}}{-2} + c$$

$$\Rightarrow z = \frac{1}{2} + ce^{2x} \quad \text{——— (4)}$$

STEP:-6

Putting the value of $z = y^{-1}$ in equation (4)

$$\boxed{y^{-1} = \frac{1}{2} + ce^{2x}}$$

37. $y' + xy = xy^{-1}$

$$\Rightarrow \frac{dy}{dx} + xy = xy^{-1} \quad \text{——— (1)}$$

which is a Bernoulli's DE with $P(x) = x$, $Q(x) = x$

STEP:-1 Dividing both sides by y^{-1}

$$y\frac{dy}{dx} + xy^2 = x \quad \text{——— (2)}$$

Scanned by CamScanner

STEP:-2

Put $y^2 = z$

$$\Rightarrow 2y \frac{dy}{dx} = \frac{dz}{dx}$$

$$\Rightarrow y \frac{dy}{dx} = \frac{1}{2} \frac{dz}{dx}$$

STEP:-3

Putting these values in equation (2)

$$\frac{1}{2} \frac{dz}{dx} + xz = x$$

$$\Rightarrow \frac{dz}{dx} + 2xz = 2x \quad \text{——— (3)}$$

STEP:-4

which is a linear DE.

So, the integrating factor is $e^{\int 2x\,dx} = e^{x^2}$

STEP-5

Multiplying the I.F into both sides of equation (3)

$$\frac{dz}{dx} e^{x^2} + 2xe^{x^2} z = 2xe^{x^2}$$

$$\Rightarrow \frac{d}{dx}(ze^{x^2}) = 2xe^{x^2}$$

$$\Rightarrow d(ze^{x^2}) = 2xe^{x^2} dx$$

Integrating both sides

$$\int d(ze^{x^2}) = \int 2xe^{x^2} dx$$

$$\Rightarrow ze^{x^2} = e^{x^2} + c$$

$$\Rightarrow z = 1 + ce^{-x^2} \quad \text{——— (4)}$$

STEP:-6

Putting the value of $z = y^2$ in equation (4)

$$\boxed{y^2 = 1 + ce^{-x^2}}$$

Scanned by CamScanner

STEPS FOR SOLVING A FIRST ORDER DE

STEP:-1

First see that equation can be solved by using variable separable or not.

STEP:-2

If not try whether that equation can be solved by using method of substitution or not.

STEP:-3

If not see that DE is a

(i) Homogeneous DE

(II) Linear DE

(III) Bernoulli's DE

If it belongs to either of the form then apply the corresponding method for solution.

STEP:-4

If not see that DE is an exact DE or not. If it is then apply the exact DE method to get the solution.

STEP:-5

If not finally find out the integrating factor & solve.

Scanned by CamScanner